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Analysis of Thermal Energy Storage System with Conjugate Turbulent Forced Convection

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A thermal energy storage system, comprised of a hollow cylinder of phase change material (PCM), with a transfer fluid pumped through the interior, was studied by a semianalytical method. The melting process in the PCM and turbulent-forced convective heat transfer inside the tube were solved simultaneously. A method of determining the local Nusselt number for various transfer fluids is discussed.

Nomenclature

A_n = constant
c = specific heat, J/kgK or constant in Eq. (20)
D = i.d. of the tube, $2r_i$, m
Fo = Fourier number, $\alpha_p t/r_i^2$
f_m = Moody friction factor, $\Delta P/[\frac{1}{2}\rho U_m(L/D)]$
G = constant in Eqs. (19) and (20)
H = latent heat, J/kg
h = convective heat transfer coefficient, W/m²K
K_f = dimensionless thermal conductivity of transfer fluid, k_f/k_p
k = thermal conductivity, W/mK
L = length of the tube, m
M = section number of the tube along the axial direction
N = section number of the PCM along the axial direction
Nu = local Nusselt number, hD/k_f
Pe = Peclet number, $RePr$
Pr = Prandtl number, ν/α_f
Pr_t = eddy Prandtl number, $\varepsilon_m/\varepsilon_t$
R = dimensionless coordinate along the radial direction, r/r_i
Re = Reynolds number, $U_m D/\nu$
r = coordinate along the radial direction, m
S = dimensionless solid–liquid interface radius, s/r_i
Ste = Stefan number, $c_p(T^0_{in} - T^0_m)/H$
s = solid–liquid interface radius, m
T = dimensionless temperature, $(T^0 - T^0_m)/(T^0_{in} - T^0_m)$
T^0 = temperature, K
t = time, s
U = dimensionless velocity, u/U_m
U_m = average velocity, m/s
u = velocity, m/s
X = dimensionless coordinate along axial direction, x/D
x = coordinate along axial direction, m
α = thermal diffusivity, m²/s
ε_m = eddy viscosity, m²/s
ε_t = eddy thermal diffusivity, m²/s
λ_n = eigenvalues
ν = kinemetic viscosity, m²/s
ρ = density, kg/m³

Subscripts
b = bulk
f = transfer fluid
i = inside radius of the tube, or grid point in axial direction
in = inlet
o = outer wall of thermal energy storage system
p = PCM
t = turbulent
w = container wall

I. Introduction

INTEREST in studying phase-change thermal energy storage systems is growing due to the phase change material (PCM) large latent heat, mulling it an efficient way to rapidly absorb or release thermal energy. The use of a hollow cylinder of PCM (similar to that in Fig. 1), as a solar energy storage system was modeled by Solomon et al.[1] Cao and Faghri[2,3] have obtained numerical solutions for this system by transient PCM/forced convection conjugate analysis. The convection inside the tube occurred in the thermal and velocity entry regions. According to their conclusions, if a steady-state fully developed correlation was used to calculate the convective heat transfer coefficient inside the tube, a significant error could be introduced. Bellecci and Conti[4] studied a similar problem. According to their conclusions, if liquid metal was used as the transfer fluid, the convective heat transfer coefficient could be determined by steady-state numerical result of the thermal and velocity entry regions.[5] If a fluid with a moderate Prandtl number was used as the transfer fluid, the

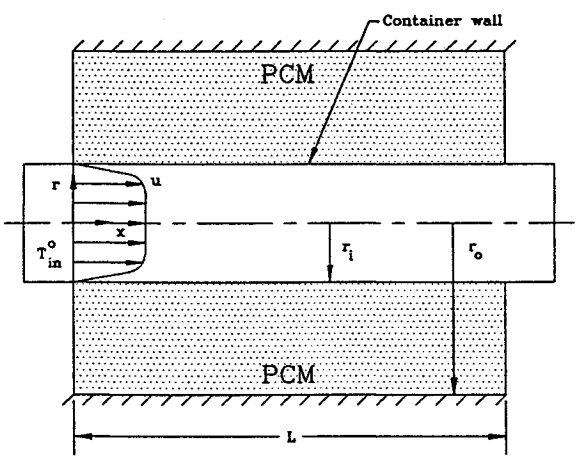

Fig. 1 Schematic of PCM thermal energy storage system with turbulent flow.

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convective heat transfer coefficient can be easily represented by fully developed correlations, and no significant errors would be introduced by this simplification.

The present authors have studied a thermal energy storage system with conjugate laminar forced convection using water as the transfer fluid by an analytical method.[6] The laminar convective heat transfer inside the tube was assumed to be in the thermal entry region while the velocity was assumed fully developed at the tube entrance. The results showed that the local Nusselt number inside the tube cannot be determined by steady-state correlations, except for very long tubes, which have no practical significance.

In energy storage systems, large amounts of heat must be transferred between the PCM and the transfer fluid. With this operating condition, the flow rate of the transfer fluid must be high and the flow turbulent in most cases. In this article, a thermal energy storage system with conjugate turbulent forced convection is investigated by an analytical method. The velocity distribution inside the tube is fully developed at the tube entrance, but the convective heat transfer in the tube is assumed to be in the thermal entry region. A method of determining the local Nusselt numbers for different transfer fluids is also discussed.

II. Physical Model

The physical model of the thermal energy storage system is shown in Fig. 1. It consists of a hollow cylinder of phase change material with a transfer fluid pumped through the interior tube for the purpose of heat exchange, with the outer surface of the hollow cylinder taken as adiabatic. In order to solve this problem, the following assumptions are necessary:

1) The inlet velocity is fully developed but heat transfer occurs in the thermal entry region.

2) Peclet numbers are larger than 100, so that axial conduction of the transfer fluid can be neglected.

3) The quasisteady assumption is applied to convective heat transfer inside the tube. Therefore, the temperature distribution in the transfer fluid depends only on the boundary conditions of the tube, but is not affected by the temperature distribution of the transfer fluid at a previous time. In other words, transient convection in the tube is treated as a series of steady-state forced convection problems.

4) Melting of the PCM in the thermal energy storage system is a two-dimensional problem. Since the variation of the container wall temperature along the axial direction is not very significant, it is assumed that axial conduction in the PCM is negligible.

5) The initial temperature is assumed to be the freezing temperature of the PCM.

6) The tube wall is very thin so that its thermal resistance can be neglected. Thus, it is assumed that the transfer fluid is directly in contact with the PCM and the thickness of tube wall is equal to zero.

After the previous assumptions, forced convection in the tube can be treated as a series of steady-state convection problems. The energy equation and the boundary conditions that describe incompressible turbulent flow with no viscous dissipation in the tube are[7]

$$u\frac{\partial T_f^0}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[(\alpha_f + \varepsilon_t)r\frac{\partial T_f^0}{\partial r}\right] \tag{1}$$

$$T_f^0 = T_i^0, \qquad x = 0 \tag{2}$$

$$T_f^0 = T_w^0, \qquad r = r_i \tag{3}$$

where the velocity distribution in the tube and eddy thermal diffusivity ε_t in Eq. (1) can be determined as given by Ref. 8.

The melting of the PCM can be treated as a series of one-dimensional melting problems in the radial direction. The energy equation of the PCM and its boundary conditions are

$$\frac{\partial T_p^0}{\partial t} = \alpha_p\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_p^0}{\partial r}\right) \tag{4}$$

$$k_p\frac{\partial T_p^0}{\partial r} = h(T_w^0 - T_b^0), \qquad r = r_i \tag{5}$$

$$T_p^0 = T_m^0, \qquad r = s < r_o \tag{6}$$

$$-k_p\left.\frac{\partial T_p^0}{\partial r}\right|_{x=s} = \rho H\frac{\mathrm{d}s}{\mathrm{d}t}, \qquad r = s < r_o \tag{7}$$

$$\left.\frac{\partial T_p^0}{\partial r}\right|_{r=r_o} = 0, \qquad r = s = r_o \tag{8}$$

By defining the following dimensionless variables:

$$R = \frac{r}{r_i} \quad S = \frac{s}{r_i} \quad R_o = \frac{r_o}{r_i} \quad K_f = \frac{k_f}{k_p} \quad U = \frac{u}{U_m} \quad X = \frac{x}{D}$$

$$Fo = \frac{\alpha_p t}{r_i^2} \quad Re = \frac{U_m D}{\nu} \quad Pe = RePr \quad Nu = \frac{hD}{k_f} \tag{9}$$

$$Ste = \frac{c_p(T_{\text{in}}^0 - T_m^0)}{H} \quad T = \frac{T^0 - T_m^0}{T_{\text{in}}^0 - T_m^0}$$

the governing equations become

$$U\frac{\partial T_f}{\partial X} = \frac{2}{Pe}\frac{1}{R}\frac{\partial}{\partial R}\left[\left(1 + Pr_t Pr\frac{\varepsilon_m}{\nu}\right)R\frac{\partial T_f}{\partial R}\right] \tag{10}$$

$$T_f = T_w, \qquad R = 1 \tag{11}$$

$$T_f = 1, \qquad X = 0 \tag{12}$$

$$\frac{\partial T_p}{\partial F_o} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T_p}{\partial R}\right) \tag{13}$$

$$\frac{\partial T_p}{\partial R} = \frac{1}{2}K_f Nu(T_w - T_b), \qquad R = 1 \tag{14}$$

$$T_p = 0, \qquad R = S < R_o \tag{15}$$

$$\frac{\mathrm{d}S}{\mathrm{d}Fo} = -Ste\frac{\partial T_p}{\partial R}, \qquad R = S < R_o \tag{16}$$

$$\frac{\partial T_p}{\partial R} = 0, \qquad R = S = R_o \tag{17}$$

III. Solutions

Forced convective heat transfer in the tube should be treated as an arbitrarily varied wall temperature forced convection problem. The full length of the tube can be divided into N sections. It can be assumed that every section has a uniform temperature. The present analysis uses the local Nusselt number for forced convective heat transfer in the tube,[6] which can be expressed as

$$Nu(X) = \frac{\sum_{i=1}^{j}\Delta T_{w,i}\sum_{n=0}^{\infty}A_n\exp\left\{-\frac{2\lambda_n^2}{Pe}[X - (i-1)\Delta X]\right\}}{2\sum_{i=1}^{j}\Delta T_{w,i}\sum_{n=0}^{\infty}\frac{A_n}{\lambda_n^2}\exp\left\{-\frac{2\lambda_n^2}{Pe}[X - (i-1)\Delta X]\right\}} \tag{18}$$

For turbulent flow in a tube, the results have the same form as the case with laminar flow. The only difference is that the eigenvalues λ_n and the values of the constant A_n depend on both the Reynolds and Prandtl numbers. The first three or four values of λ_n and A_n can be found in Refs. 8 or 9. In order to calculate the local Nusselt number at an arbitrarily varied wall temperature, additional values of λ_n and A_n are needed. Asymptotic values for the additional eigenvalues λ_n and the constant A_n can be calculated by the following formulas[8]:

$$\lambda_n = (1/G)(n + \tfrac{2}{3}) \tag{19}$$

$$A_n = (0.201/G)(Ref_m/32\lambda_n)^{1/3}\{1 - (1/G^2\lambda_n^2) \times [(c/2\pi)(\sin G\lambda_n\pi - 1) + (7/36\pi^2)]\} \tag{20}$$

The constants G and c in Eqs. (19) and (20) can be found in Ref. 8, and the Moody friction factor f_m can be found in Ref. 10. The value of j in Eq. (18) can be determined by

$$j = \text{int}(X/\Delta X) + 1 \tag{21}$$

where int in Eq. (21) is an integer function.

The dimensionless bulk temperature of the transfer fluid can be obtained by the energy conservation equation for the transfer fluid

$$Nu(T_w - T_b) = \frac{1}{4} Pe \frac{\partial T_b}{\partial X} \tag{22}$$

The full length of the PCM can be divided into M sections. Since axial conduction in the PCM has been neglected, the heat transfer in the PCM can be treated as a series of radial, one-dimensional problems. Due to the existence of an adiabatic shell, melting in the PCM occurs in a finite region, and the heat transfer in the PCM can be divided into two stages. Before the solid–liquid interface reaches the shell, a melting process is occurring in the PCM. After the solid–liquid interface reaches the shell, the melting process will stop and heat transfer in the PCM becomes a simple conduction problem. Heat conduction in the PCM, with and without phase change, can be solved by an integral approximation method.[6]

For the melting process, the temperature distribution in the PCM is

$$T_p = T_w + \varphi(\sin R/\sin S) - (T_w + \varphi)(\sin R/\sin S)^2 \tag{23}$$

where T_w is the dimensionless tube wall temperature, and φ can be expressed as follows:

$$2T_w + \varphi = \frac{\sqrt{1 + 2T_w Ste} - 1}{Ste} \tag{24}$$

The location of solid–liquid interface can be expressed as

$$\frac{dS}{dFo} = \frac{\sqrt{1 + 2T_w Ste} - 1}{S \sin S} \tag{25}$$

Substituting Eq. (23) into Eq. (14), the equation for the dimensionless tube wall temperature can be obtained

$$\frac{1}{2} K_f Nu(T_b - T_w) = \frac{1}{\sin S}\left(2T_w - \frac{\sqrt{1 + 2T_w Ste} - 1}{Ste}\right) \tag{26}$$

After the solid–liquid interface reaches the outer container wall, heat transfer in the PCM will become a pure conduction problem with a boundary condition of the third kind at the inner boundary ($R = 1$) and an adiabatic boundary condition at the outer boundary ($R = R_o$). Therefore, the temperature profile in the liquid PCM is

$$T_p = T_w - 2\eta(\sin R/\sin R_o) + \eta(\sin R/\sin R_o)^2 \tag{27}$$

where η can be obtained from an integral equation[6]

$$\left[\frac{R_o^2 - 1}{4(\sin R_o)^2} - \frac{1}{2\sin R_o} - \frac{R_o^2}{2}\right]\frac{d\eta}{dFo} - \frac{2}{\sin R_o}\eta + \frac{R_o^2 - 1}{2}\frac{dT_w}{dFo} = 0 \tag{28}$$

The initial condition for η must satisfy the total sensible heat of the PCM, varied continually with time. If it can be assumed that the solid–liquid interface reached the shell at $Fo = Fo_o$, the initial value of η can be determined by the following equations[11]:

$$\int_{R_w}^{R_o} RT_p(R, Fo_o^-)\, dR = \int_{R_w}^{R_o} RT_p(R, Fo_o^+)\, dR \tag{29}$$

where $T_p(R, Fo_o^-)$ and $T_p(R, Fo_o^+)$ in Eq. (29) can be calculated by Eqs. (23) and (27), respectively.

Substituting Eq. (27) into Eq. (14), the equation of dimensionless tube wall temperature can be obtained

$$\tfrac{1}{2}K_f Nu(T_b - T_w) = 2\eta/\sin R_o \tag{30}$$

The calculation is started from $Fo = 0$. For any time step, the problem can be solved by the following procedure:

1) Estimate a tube wall temperature distribution along the axial direction $T_w(X)$.

2) Calculate the location of the solid–liquid interface or η for different locations of the axial coordinate X.

3) Calculate the local Nusselt number Nu by Eq. (18) for different locations of the axial coordinate X.

4) Solve Eq. (26) or Eq. (30) to obtain the variation of the tube wall temperature $T'_w(X)$.

5) Compare the assumed wall temperature $T_w(X)$ and the calculated wall temperature $T'_w(X)$. If $|T_w - T'_w|_{\max} \le 10^{-5}$, calculate the next time step, if not, return to step 1.

During the iteration of T_w, underrelaxation is necessary. The relaxation factor used was 0.8. It was found that the effect of N is stronger than the effect of M on the final result. After this evaluation, the grid size used for the calculation was $N = 50$ and $M = 150$ and the dimensionless time step was $\Delta Fo = 0.05$.

IV. Results and Discussion

Convective heat transfer inside the tube and the melting of the PCM are coupled by Eq. (14). The boundary condition for the melting process depends on the product of dimensionless thermal conductivity of the transfer fluid and the local Nusselt number.

Figures 2 and 3 show the variations of local Nusselt number along the axial direction for varying dimensionless thermal conductivity values for the liquid metal transfer fluid at $Re = 5 \times 10^5$ and $Pr = 0.004$. The local Nusselt number for turbulent forced convection under the boundary conditions of constant wall temperature (CWT) and constant heat flux (CHF) at the same Reynolds number and Prandtl number[8] are given in Figs. 2 and 3 for comparison. The variations in local Nusselt number are strongly dependent on the value of K_f. For $K_f = 0.1$, the local Nusselt number is heavily equal to that for the CWT boundary condition at the beginning of the melting process, which slightly increases with time. Compared to the variation of the local Nusselt number with the CHF boundary condition, the variation of local Nusselt number in a hollow cylinder of PCM is closer to the local Nusselt

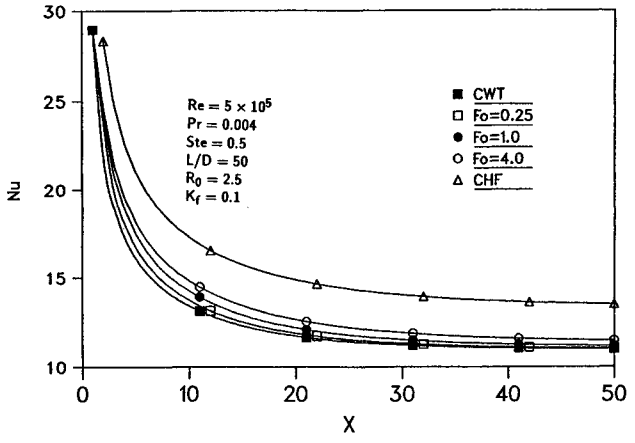

Fig. 2 Local Nusselt number for liquid metal transfer fluid (K_f = 0.1).

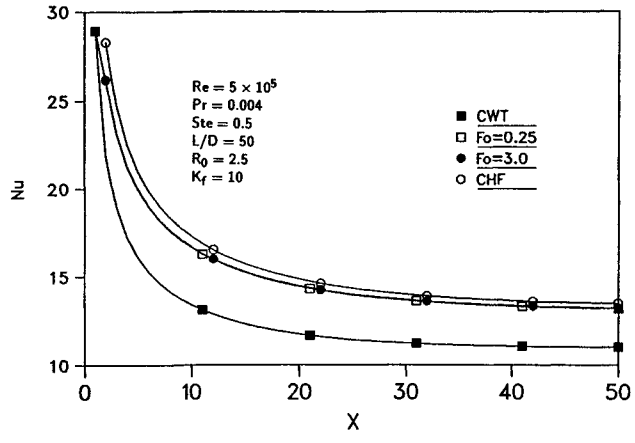

Fig. 3 Local Nusselt number for liquid metal transfer fluid (K_f = 10).

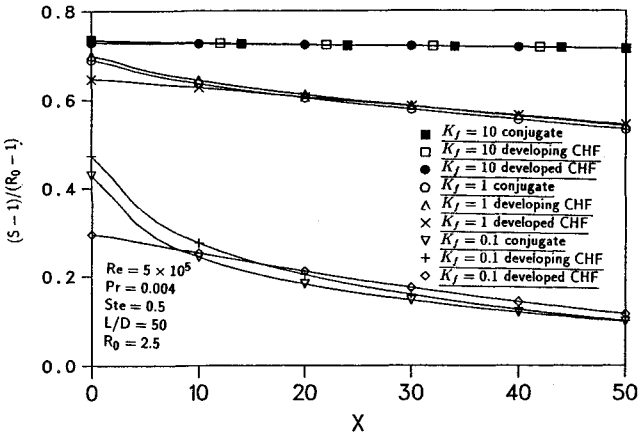

Fig. 4 Solid–liquid interface for liquid metal transfer fluid (Fo = 2.0).

number with the CWT boundary condition. For K_f = 10, the local Nusselt number is almost unchanged with time, and is very similar to the local Nusselt number associated with the CHF boundary condition.

Figure 4 shows the solid–liquid interface locations for different dimensionless thermal conductivities of the transfer fluid at Fo = 2. The solid–liquid interface locations calculated by using the thermally developing and fully developed local Nusselt number under the CHF boundary condition are also given in Fig. 4. For K_f = 0.1, if the local Nusselt number is calculated by using the fully developed turbulent correlation (i.e., local Nusselt number will not change along the axial direction), a significant error will be introduced. Bellecci and Conti[4] suggested that the local Nusselt number can be determined in the entry region using the CHF boundary condition. The melting rate calculated by Bellecci and Conti's suggestion is greater than that predicted by the conjugate analysis. In fact, it can be shown in Fig. 2 that the variation of local Nusselt number at lower K_f is not closer to the local Nusselt number with the CHF boundary condition. Therefore, for both lower Prandtl number and lower K_f, the local Nusselt number in Eq. (14) cannot be determined by the steady-state thermally developing or developed Nusselt number. For K_f = 1, although the local Nusselt number also cannot be approximated as a fully developed result under the CHF boundary conditions, Bellecci and Conti's suggestion is accurate for the solid–liquid interface locations. For K_f = 10, the solid–liquid interface locations obtained by the three different methods are almost identical. A significant error will not be introduced if the local Nusselt number is calculated by the fully developed result under the CHF boundary condition.

Cao and Faghri[3] studied a similar problem, but the convective heat transfer in the tube occurred in the thermal and velocity entry region. According to their result for Re = 6.095 × 10^5, Pr = 0.05, and K_f = 0.1094, the local Nusselt number cannot be calculated by the fully developed turbulent correlation. This conclusion is confirmed by the present article. However, a very low Prandtl number means that a liquid metal with very high thermal conductivity is used as the transfer fluid. If a liquid metal was used as the transfer fluid, the thermal conductivity of the transfer fluid should be larger than the thermal conductivity of the PCM. Therefore, the dimensionless thermal conductivity of the transfer fluid will not be less than 1. For example, the dimensionless thermal conductivity of a liquid metal transfer fluid used in Cao and Faghri's analysis was K_f = 11.5.[2] According to Fig. 4, it is not necessary to determine the local Nusselt number by the present conjugated analysis for this value of dimensionless thermal conductivity. Therefore, even for a liquid metal transfer fluid, the local Nusselt number in Eq. (14) can be determined by the steady-state fully developed result under the CHF boundary condition for most circumstances, unless the thermal conductivity of the PCM is larger than the liquid metal.

Figure 5 shows the variation of the local Nusselt number along the axial direction at different times for Re = 10^5 and Pr = 0.72. Local Nusselt number for turbulent forced convection under the CWT and CHF boundary condition at the same Reynolds number and Prandtl number is also given in Fig. 5 for comparison. The values of the local Nusselt numbers are very close to those of the local Nusselt numbers with the CHF boundary condition. Figure 6 shows the solid–liquid

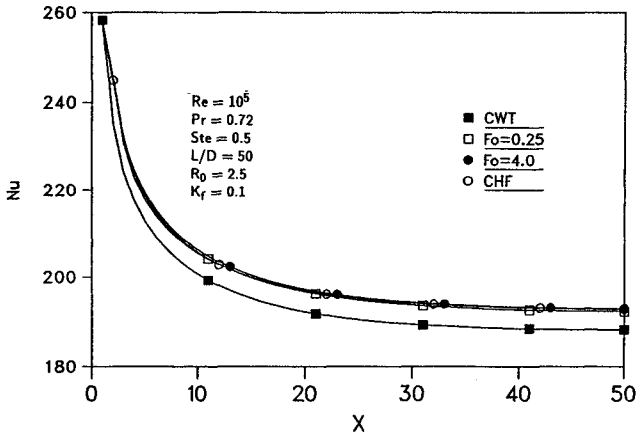

Fig. 5 Local Nusselt number for Pr = 0.72.

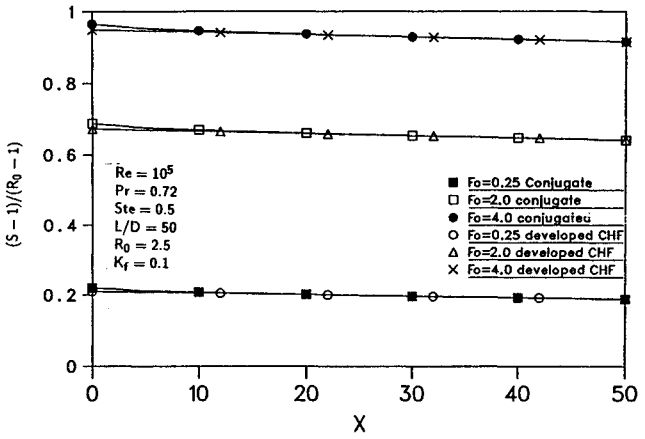

Fig. 6 Solid–liquid interface for $Pr = 0.72$.

interface location at different times for $Re = 10^5$ and $Pr = 0.72$. The solid–liquid interface location calculated by using the fully developed CHF correlation is also given in Fig. 6 for comparison. For the transfer fluid with $Pr = 0.72$, the local Nusselt number can be calculated by using the fully developed correlations and a significant error will not be introduced. In fact, the effects of the boundary conditions on the local Nusselt number are not significant for the turbulent internal flow at $Pr \geq 0.5$.[7] Therefore, the difference between the local Nusselt number with the CWT and CHF boundary conditions are not significant in Fig. 5. In other words, any variation in the wall temperature and heat flux along the axial direction will have no significant effect on the local Nusselt number for $Pr \geq 0.5$. If a fluid with a Prandtl number larger than 0.5 (such as air or water), is used as the transfer fluid, the local Nusselt number can be calculated by the fully developed correlations.

The results of present analysis showed that the local Nusselt number can be simply quoted from the fully developed correlations for most circumstances. This conclusion is completely different from the conclusion of the present authors' previous work about the analysis of thermal energy storage system with conjugate laminar forced convection.[6] The results of Ref. 6 showed that the local Nusselt number inside the tube cannot be simply quoted from that of developing heat transfer for CWT or CHF boundary conditions. The reason for such a different conclusion is that the values of local Nusselt number for laminar flow are much smaller than that for turbulent flow. Therefore, any changes on the values of local Nusselt number will result in a significant error. On the other side, the developing length of laminar heat transfer inside a tube is much longer than that of turbulent heat transfer. Therefore, present analysis indicated a different conclusion from laminar heat transfer.

V. Conclusions

A thermal energy storage system with a turbulent transfer fluid flow was studied semianalytically in this article. For most circumstances, the local Nusselt number can be calculated by the fully developed correlations with CHF boundary condition, except for both lower Prandtl number and lower dimensionless thermal conductivity of the transfer fluid. Calculations in the design of thermal energy storage systems can be significantly simplified by using the methodology presented herein.

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